Randomly Imperfect Waveguides for Millimeter and Submillimeter Wavelengths for Long-Distance Transmission Applications

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***Abstract*—Randomly imperfect waveguides are considered for use in long-distance transmission at millimeter and submillimeter wavelengths. The steady-state attenuation constant, modal power distribution, and pulse spreading are calculated from Marcuse's coupled power equations. The result shows that a random waveguide can transmit 3.6 Gbit/s per square root of kilometer with a loss of 1.7 dB/km at 500 GHz.**

Mode conversions due to circular bends are considered next. It is shown that the systematic mode conversion due to bends is not of prime importance if the random coupling coefficient is much larger than the systematic one. This may be the most important feature of random metal waveguides for long-distance transmission applications.

I. INTRODUCTION

VARIOUS transmission systems are under study for possible applications in meeting with future telecommunication demands. The digital radio systems are promising, below 30 GHz [1]. A guided millimeter-wave system can offer about 300 000 two-way voice channels using the 43–87-GHz range [2]. With the advent of the low-loss optical fibers, optical waveguide systems are considered to be most promising in the future [3]. In conventional systems, various modulation formats are used to put information on the waveform, while in certain optical systems, the information is in the intensity of the wave (optical carrier intensity modulation).

In this paper, we consider the possibility of using waveguides at millimeter and submillimeter wavelengths for long-distance transmission. Radio systems are unlikely to operate at these wavelengths because of the heavy rainfall attenuation. The guided millimeter-wave system can, in principle, be extended to even higher frequencies. However, it is difficult to fabricate such waveguides for the higher frequencies. The tolerance becomes stringent and waveguide installation will raise yet another problem. Other configurations such as the G line [4] and the dielectric waveguide may have higher losses than the metallic waveguide. The waveguide considered here is made of metal and is multimoded as in the guided millimeter-wave system. However, the modes are randomly coupled in this guide. This is reminiscent of the multimode optical waveguides [5]. The guide is thus applied to the transmission of wave intensity. The modal power distribution and the pulse spreading are important in such waveguides [6], [7]. This paper describes those in the metal waveguides, using a two-dimensional model.

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II. POWER DISTRIBUTION AND PULSE SPREADING

A. Power Distribution

The power flow in each mode characterizes random waveguides. Marcuse has developed the theory for randomly imperfect optical waveguides [8]. We apply his theory to the analysis of the present waveguide. Fig. 1 shows a two-dimensional random waveguide with the appropriate coordinate system. For simplicity, we assume that the lower plate is perfect while the upper plate is randomly distorted with a small distortion $f(z)$. The guide boundaries are thus determined by $x = 0$ and $x = a + f(z)$, where a is a mean value. An ensemble average of $f(z)$ is assumed to be Gaussian, i.e.,

$$\langle f(z)f(z-u)\rangle = \bar{\sigma}^2 \exp\left[-(u/D)^2\right] \qquad (1)$$

where $\bar{\sigma}$ and D are the rms deviation and the correlation distance of the upper plate, respectively, and $\langle\ \rangle$ denotes an ensemble average of similar guides.

The coupled power equations have been developed by Marcuse and they are written as [8], [9]

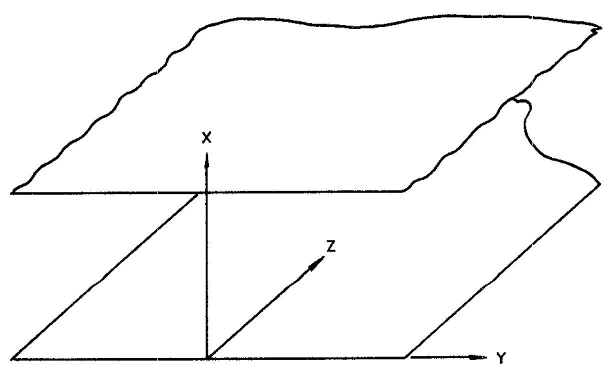

Fig. 1. A two-dimensional waveguide with a randomly imperfect wall. The guide is uniform along the y axis. The waves are propagated down the z axis. The upper plate is determined by $x = a + f(z)$, where $f(z)$ is a small distortion function.

$$\frac{dP_\nu}{dz} = -(\alpha_\nu + b_\nu)P_\nu + \sum_{\mu=1}^{N} h_{\nu\mu}P_\mu, \qquad (\nu = 1,2,\cdots,N) \tag{2}$$

where P_ν is the power carried by the mode ν, α_ν is its attenuation constant, and

$$b_\nu = \sum_{\mu=1}^{N} h_{\nu\mu}, \qquad (\nu = 1,2,\cdots,N) \tag{3}$$

$h_{\nu\mu}$ represents the power coupling coefficient between the mode ν and μ. For simplicity, we consider only the TE modes. In this case, the attenuation constant is obtained from Maxwell's equations as follows:

$$\alpha_\nu = \frac{2\delta}{\beta_\nu a}\left(\frac{\nu\pi}{a}\right)^2, \qquad (\nu = 1,2,\cdots,N) \tag{4}$$

where δ is the skin depth of the metal and β_ν is given by

$$\beta_\nu = \left[k^2 - \left(\frac{\nu\pi}{a}\right)^2\right]^{1/2} \tag{5}$$

where k is the free-space wavenumber.

The amplitude coupling coefficient is derived from Schelkunoff's normal mode theory [10]. The resulting coupled equations are written in terms of an equivalent voltage and current. We then rewrite them in the form of forward- and backward-traveling waves. Neglecting backward waves, the amplitude coupling coefficient is obtained as

$$K_\nu{}^\mu = \frac{\nu\mu(-1)^{\nu+\mu}}{\mu^2 - \nu^2}\left[(\beta_\mu/\beta_\nu)^{1/2} + (\beta_\nu/\beta_\mu)^{1/2}\right]\frac{1}{a}\frac{df(z)}{dz}. \tag{6}$$

In order to evaluate the power coupling coefficient $h_{\nu\mu}$ from (6), it is necessary to calculate an ensemble average of $df(z)/dz$. The assumed Gaussian autocorrelation enables us to calculate it in the closed form. Its derivation is shown in the Appendix. The result is given by

$$\left\langle \frac{df(z)}{dz}\frac{df(z-u)}{dz}\right\rangle = -4\bar{\sigma}^2 D \frac{\partial}{\partial(D^2)}\left[\frac{\exp[-(u/D)^2]}{D}\right]. \tag{7}$$

Using (6) and (7), we can write the power coupling coefficient in the following form (see Appendix):

$$h_{\nu\mu} = \left\{\frac{\nu\mu}{\mu^2 - \nu^2}\left[(\beta_\mu/\beta_\nu)^{1/2} + (\beta_\nu/\beta_\mu)^{1/2}\right]\right\}^2 \cdot \frac{\bar{\sigma}^2 D}{a^2}(\beta_\nu - \beta_\mu)^2\pi^{1/2}\exp\{-[\tfrac{1}{2}D(\beta_\nu - \beta_\mu)]^2\}. \tag{8}$$

Following Marcuse [11], we can calculate the steady-state attenuation constants $\alpha^{(i)}$ and eigenvectors $B_\nu{}^{(i)}$ from (2), (4), and (8). The term $\alpha^{(1)}$ represents the lowest order attenuation constant of the coupled system. Hence a sufficiently long waveguide distributes the power $B_\nu{}^{(1)}$ to the mode ν ($\nu = 1,2,\cdots,N$), and the coupled dominant wave is attenuated at the rate of $\alpha^{(1)}$.

Fig. 2 shows the normalized attenuation constants $\alpha^{(1)}a$ and $\alpha^{(2)}a$ as functions of D/a. The solid lines show the case with $N = 40$, and the dot–dash line shows the case with $N = 20$; the broken lines depict $\alpha^{(2)}a$. In this paper we assume that the metal is silver whose conductivity is 6.139×10^4 mho/mm. It is apparent from (8) that $h_{\nu\mu}$ vanishes when D/a becomes very large or very small, as in optical waveguides [11]. However, α_ν is independent of D/a. Hence each eigenvalue derived from the determinant of the coupled system approaches α_ν when D/a becomes very large or very small. The lowest eigenvalue $\alpha^{(1)}$ ultimately coincides with α_1 of (4). Furthermore, a large $\bar{\sigma}/a$ gives a large eigenvalue. As the mode number increases, the maxima of eigenvalues move to the points with small correlation distances. This shows that the eigenvalue is determined with the correlation distance in relation to the guide wavelength.

In optical waveguides, α_ν depends on D/a. For very

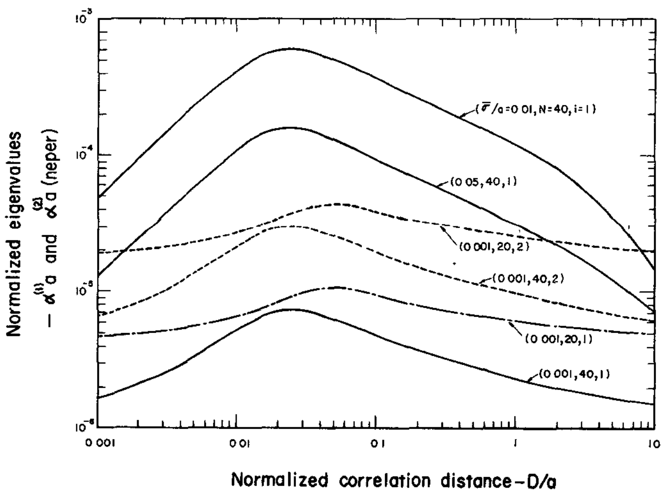

Fig. 2. The first and second eigenvalues ($i = 1,2$) obtained from the coupled power equations (2). The first eigenvalue shows the steady-state attenuation of the coupled system in a sufficiently long waveguide. The parameters N and $\bar{\sigma}/a$ are the largest mode number and the normalized rms deviation, respectively.

small D/a, α_ν is proportional to D/a, and, for very large D/a, it is proportional to $\exp\{-[\frac{1}{2}D(\beta_\nu - k_2)]^2\}$ where k_2 is the propagation constant in the surrounding medium of the guide [11]. Thus α_ν also vanishes when D/a becomes very large or very small. This does affect the power distribution in optical waveguides. For large D/a, the power tends to distribute evenly among the modes.

The power distribution of random metal waveguides is different from that of the optical waveguides. Fig. 3 shows the steady-state power distribution for $N = 40$ as a function of mode number ν. The normalized rms deviation $\bar{\sigma}/a$ is assumed to be 0.001. Contrary to the optical waveguides, the most power resides in the dominant mode even when D/a is very large. One could argue that the assumed $\bar{\sigma}/a$ is too small to cause enough mode coupling. However, it is characteristic of random metal waveguides that several lower order modes carry most of the power no matter what the correlation distance is assumed to be. It is indeed verified with Fig. 4, where $\bar{\sigma}/a$ is assumed to be 0.01. This gives a large loss penalty as seen from Fig. 2.

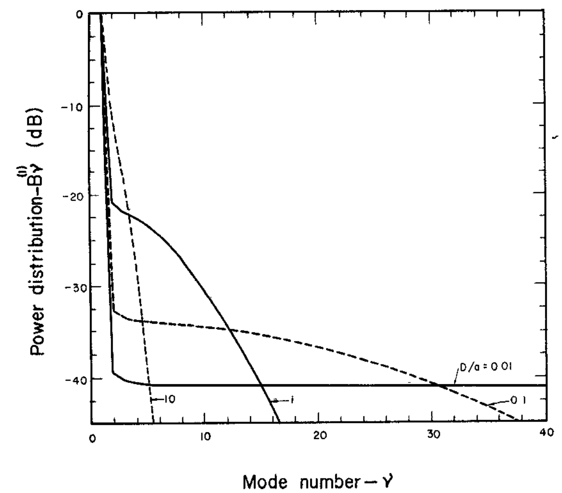

Fig. 3. The first eigenvector obtained from (2). $B_\nu^{(1)}$ shows the power in mode ν in a sufficiently long waveguide. The deviation $\bar{\sigma}/a$ is assumed to be 0.001.

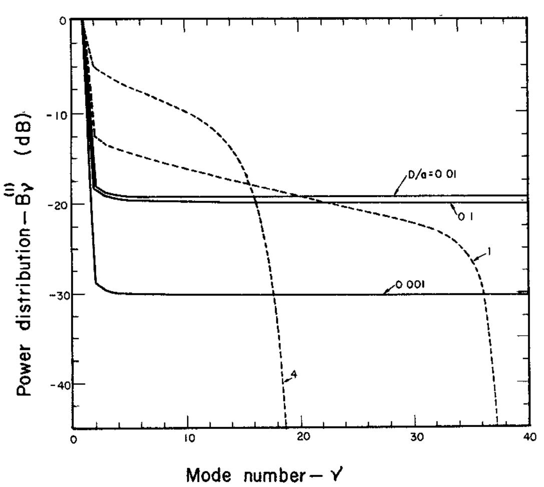

Fig. 4. The power distribution as functions of mode number ν, where $\bar{\sigma}/a$ is assumed to be 0.01.

One question comes to mind though. Marcuse has shown that if the loss difference between a pair of modes is larger than the coupling coefficient, the pulsewidth increases with increasing coupling strength [8]. Hence the mode mixing is useful for reducing the pulse spreading in the case that the coupling strength is larger than the loss difference. In metal waveguides, it is also possible to reduce the pulse spreading by mode mixing. It has been shown that α_ν is independent of D/a while $h_{\nu\mu}$ depends on D/a. Hence we can make $h_{\nu\mu}$ either large or small compared to the difference between α_ν and α_μ with the suitable choice of D/a or $\bar{\sigma}/a$. The result on the pulse spreading will be shown later.

Fig. 5 shows how the steady-state power distribution is attained by changing the guide length. It is shown that if the normalized length z/a is $10^5 \sim 10^6$, we get the steady-state distribution with equal modal power excitation at $z = 0$.

B. Pulse Spreading

The second-order perturbation solution of (2) enables us to calculate the pulse spreading. If a sufficiently narrow pulse is transmitted, its spreading is calculated from [12]

$$t = 4(\alpha_2^{(1)}L)^{1/2} \tag{9}$$

where $\alpha_2^{(1)}$ is the second-order eigenvalue of the time-dependent system. Fig. 6 shows the pulsewidth as a function of D/a. It is normalized by $(a)^{1/2}$ so that the width at the distance L is obtained from the ordinate value times $(L/a)^{1/2}$. The solid lines show the case of $N = 40$, and the broken line shows $N = 20$. For small D/a, the width is also small because of the small power coupling. It increases with increasing D/a. This coincides with Marcuse's theory for the coupling strength that is smaller than the loss difference. The width then reaches the maximum and decreases with increasing D/a. This also coincides with his theory. This is because, as stated before, $h_{\nu\mu}$ depends on D/a while α_ν is independent of D/a. This region thus corresponds to the case in which the coupling strength is larger than the loss difference. Hence mode mixing is indeed useful for reducing the pulse spreading in random metal waveguides. After reaching the minimum point, the width increases with increasing D/a. This again shows the region for the coupling strength that is smaller

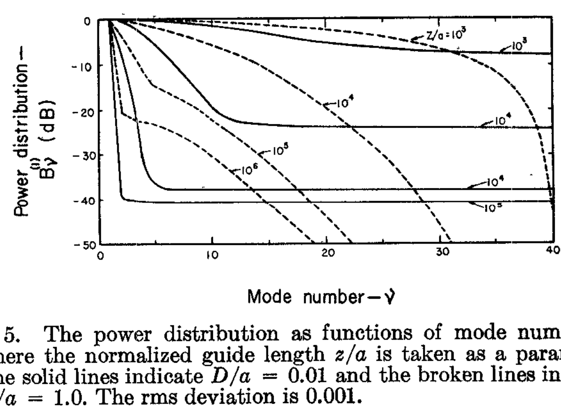

Fig. 5. The power distribution as functions of mode number ν, where the normalized guide length z/a is taken as a parameter. The solid lines indicate $D/a = 0.01$ and the broken lines indicate $D/a = 1.0$. The rms deviation is 0.001.

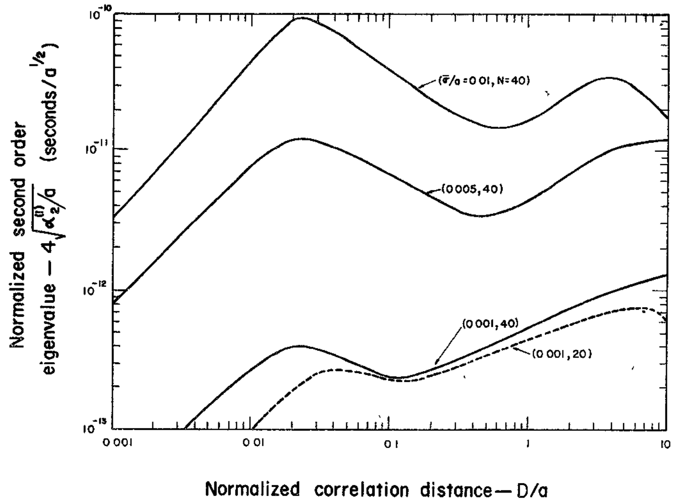

Fig. 6. The normalized second-order solution of the time-dependent coupled power equations. The pulsewidth at the distance L is obtained by multiplying the ordinate with $(L/a)^{1/2}$, where a is the plate separation.

than the loss difference. For very large D/a, the eigenvalue is split into the attenuation constant of each mode. Therefore, the pulsewidth should decrease with very large D/a. However, as he claims [8], Marcuse's theory does not seem to apply to this case.

We conclude this section after showing an example. We assume the frequency is 500 GHz and guide width is 12.2 mm. Forty modes propagate in this guide. From Fig. 2, the steady-state attenuation becomes 1.7 dB/km with 1.2-mm correlation distance and 12-μm-rms deviation. Since the uncoupled dominant mode has a loss of 0.4 dB/km, the loss penalty is about 1.3 dB/km. Fig. 6 shows that the pulse rate in this case is 3.6 Gbit/s per square root of kilometer. Hence a few hundred megabits/second can be transmitted over several kilometers with small loss penalty.

III. MODE CONVERSIONS DUE TO CIRCULAR BENDS

One of the important considerations for metal waveguides is mode conversions due to bends. Bends cause systematic coupling in a well-fabricated waveguide,[1] and, in certain cases, they result in complete power exchange between modes. For example, the TE_{0n} modes in a circular waveguide are systematically converted to the TM_{1n} modes in a circular bend. In random waveguides, the systematic mode conversion is not of prime importance if the random coupling coefficient is much larger than the systematic one. This is because the constant buildup of higher modes is interrupted by the larger random coupling. This is perhaps the most important aspect of the random waveguides. Bends due to route, conduit condition, etc., are unavoidable in a long transmission line. Random waveguides may circumvent this difficulty, i.e., suffering from a large loss penalty.

Mode conversions due to circular bends are again calculated from the normal mode theory. The random and systematic coupling coefficients are obtained as

$$K_r = p_{\nu\mu} \frac{1}{a} \frac{df(z)}{dz} (\beta_\nu + \beta_\mu)$$

$$K_S = -jp_{\nu\mu} \frac{1}{r^2} \tag{10}$$

where $p_{\nu\mu}$ is a constant and r is a bending radius. Hence, if

$$\left| \frac{1}{a} \frac{df(z)}{dz} (\beta_\nu + \beta_\mu) \right| \gg \frac{1}{r^2} \tag{11}$$

the systematic coupling may be negligibly small. It has been shown that $df(z)/dz$ can be replaced by $j(\beta_\nu - \beta_\mu)f(z)$, as far as the coupling is concerned [13]. The preceding equation then yields

$$\frac{\bar{\sigma}}{a} \gg \frac{0.0716}{\mu^2 - \nu^2} \left(\frac{a}{r}\right)^2 \tag{12}$$

where $f(z)$ was replaced by $2^{1/2}\bar{\sigma}$. Equation (12) shows that a minimum bending radius of, say, 1 m is sufficient to preclude the systematic mode conversion if $\mu \neq \nu$, for the example given in the previous section. If $\mu = \nu$, it is not possible to choose $\bar{\sigma}/a$ that satisfies (12). Therefore, it is important to introduce in random waveguides some means that can split the propagation constants of degenerate modes.

IV. CONCLUSION

We have considered the feasibilities of metallic waveguides with random imperfections. The random imperfections cause mode conversions among various modes in random fashion. The power distribution and the attenuation constants of the coupled system are calculated from the coupled power equations developed by Marcuse. The pulse spreading is derived from the second-order perturbation solution to the time-dependent coupled system.

In random metal waveguides, we have two regions that distinguish mode mixing: one is the case where the pulsewidth increases with increasing coupling strength. The other is the case where the width decreases with increasing coupling strength. We thus have a minimum pulse spreading in relation to the correlation distance of random walls.

The systematic coupling coefficient of circular bends is calculated and compared to the random coupling coefficient. Because it is possible to choose the random coupling coefficient to be much larger than the systematic one,

[1] The term "systematic coupling" is used to distinguish it from random coupling. Examples are distributed directional couplers and mode transducers.

bending losses are not of prime importance in the random waveguides. This is perhaps the most important point of the random waveguides for long-distance transmission applications.

APPENDIX

If a random variable $f(z)$ is stationary and ergodic, its ensemble average is calculated from the following formula [14]:

$$R(u) = \lim_{L\to\infty} \frac{1}{L}\int_{-L/2}^{L/2} f(z)f(z-u)\,dz. \tag{13}$$

Using the Fourier transform $F(\omega)$ of $f(z)$, we can rewrite (13) as follows:

$$R(u) = \int_{-\infty}^{\infty} \frac{2\pi\,|F(\omega)|^2}{L} \exp(-j\omega u)\,d\omega \tag{14}$$

where the mathematically difficult limiting process on L is implied [14]. The inverse transform of (14) yields

$$\frac{2\pi\,|F(\omega)|^2}{L} = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(u)\exp(j\omega u)\,du. \tag{15}$$

If $R(u)$ is given by a Gaussian function (1) in the text, the power spectrum of $R(u)$ is obtained as

$$\frac{2\pi\,|F(\omega)|^2}{L} = \frac{\bar{\sigma}^2 D}{2(\pi)^{1/2}} \exp[-(\tfrac{1}{2}D\omega)^2]. \tag{16}$$

Now, based on the assumed random process, we can write

$$V(u) = \left\langle \frac{df(z)}{dz}\frac{df(z-u)}{dz} \right\rangle$$

$$= \lim_{L\to\infty}\int_{-L/2}^{L/2} \frac{df(z)}{dz}\frac{df(z-u)}{dz}\,dz. \tag{17}$$

Because $F(\omega)$ is the Fourier transform of $f(z)$, we have

$$\frac{df(z)}{dz} = \int_{-\infty}^{\infty} j\omega F(\omega)\exp(j\omega z)\,dz. \tag{18}$$

Substitution of (18) into (17) yields

$$V(u) = \int_{-\infty}^{\infty} \frac{2\pi\omega^2\,|F(\omega)|^2}{L}\exp(-j\omega u)\,d\omega. \tag{19}$$

From (16) and (19), (6) in the text is derived.

We next show the derivation of (7). The power coupling coefficient is written as

$$h_{\nu\mu} = |k_{\nu\mu}|^2 \int_{-\infty}^{\infty} \left\langle \frac{df(z)}{dz}\frac{df(z-u)}{dz} \right\rangle \cdot \exp[j(\beta_\nu - \beta_\mu)u]\,du \tag{20}$$

where

$$|k_{\nu\mu}|^2 = \left\{\frac{\mu\nu}{\mu^2-\nu^2}[(\beta_\nu/\beta_\mu)^{1/2} + (\beta_\mu/\beta_\nu)^{1/2}]\right\}^2 \frac{1}{a^2}. \tag{21}$$

Using (6) in the text, we can calculate the integral as follows:

$$-4\bar{\sigma}^2 D\int_{-\infty}^{\infty}\frac{\partial}{\partial(D^2)}\left(\frac{\exp[-(u/D)^2]}{D}\right) \cdot \exp[j(\beta_\nu-\beta_\mu)u]\,du$$

$$= -4\bar{\sigma}^2 D\frac{\partial}{\partial(D^2)} \cdot \left\{\frac{1}{D}\int_{-\infty}^{\infty}\exp[-(u/D)^2 + j(\beta_\nu-\beta_\mu)u]\,du\right\}$$

$$= -4\bar{\sigma}^2 D\frac{\partial}{\partial(D^2)}\left[\pi^{1/2}D\cdot\frac{\exp\{-[\frac{1}{2}D(\beta_\nu-\beta_\mu)]^2\}}{D}\right]$$

$$= \bar{\sigma}^2 D\pi^{1/2}(\beta_\nu-\beta_\mu)^2\exp\{-[\tfrac{1}{2}D(\beta_\nu-\beta_\mu)]^2\}. \tag{22}$$

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REFERENCES

[1] H. Yamamoto *et al.*, "Experimental 20 GHz multi-hop digital radio-relay link," NTT Tech. Rep. 72-0J, Apr. 1974, pp. 1–8.
[2] S. Seki *et al.*, "Transmitter-receivers for the millimeter-wave transmission system in the 43–87 GHz range," in *Proc. 4th European Microwave Conf.*, paper C-9 (Sept. 1974), pp. 624–628.
[3] S. E. Miller, E. A. J. Marcatili, and T. Li, "Research toward optical-fiber transmission systems—Part I: The transmission medium," *Proc. IEEE* (*Special Issue on Minicomputers*), vol. 61, pp. 1703–1726, Dec. 1973.
S. E. Miller, T. Li, and E. A. J. Marcatili, "Part II: Devices and Systems Considerations," *ibid.*, vol. 61, pp. 1726–1751, Dec. 1973.
[4] G. Goubau, "Surface waves and their applications to transmission lines," *J. Appl. Phys.*, vol. 21, pp. 1119–1128, Nov. 1950.
[5] S. D. Personick, "Time dispersion in dielectric waveguides," *Bell Syst. Tech. J.*, vol. 50, pp. 843–850, Mar. 1971.
[6] D. Gloge, "Optical power flow in multimode fibers," *Bell Syst. Tech. J.*, vol. 51, pp. 1767–1783, Oct. 1972.
[7] ——, "Impulse response of clad optical multimode fibers," *Bell Syst. Tech. J.*, vol. 52, pp. 801–816, Jul.–Aug. 1973.
[8] D. Marcuse, *Theory of Dielectric Optical Waveguides.* New York: Academic, 1974, ch. 5.
[9] ——, "Derivation of coupled power equations," *Bell Syst. Tech. J.*, vol. 51, pp. 229–237, Jan. 1972.
[10] S. A. Schelkunoff, "Conversion of Maxwell's equations into generalized telegraphist's equations," *Bell Syst. Tech. J.*, vol. 34, pp. 995–1043, Sept. 1955.
[11] D. Marcuse, "Power distribution and radiation losses in multimode dielectric slab waveguides," *Bell Syst. Tech. J.*, vol. 51, pp. 429–454, Feb. 1972.
[12] ——, "Pulse propagation in multimode dielectric waveguides," *Bell Syst. Tech. J.*, vol. 51, pp. 1199–1232, Jul.–Aug. 1972.
[13] ——, "Coupled mode theory of round optical fibers," *Bell Syst. Tech. J.*, vol. 52, pp. 817–842, Jul.–Aug. 1973.
[14] For example, see W. R. Bennett, "Methods of solving noise problems," *Proc. IRE*, vol. 44, pp. 609–638, May 1956.